Lecture 2: Dynamo theory

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Basis of electromagnetism: Maxwell’s equations

- Faraday’s law of induction: if a magnetic field $\mathbf{B}$ varies with time then an electric field $\mathbf{E}$ is produced.

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}
\]

- Ampère’s law (velocity $\ll$ speed of light)

\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{j}
\]

where $\mathbf{j}$ is the current density and $\mu_0$ is the vacuum magnetic permeability.

- Gauss’s law (electric monopoles from which electric field originates)

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}
\]

with $\rho$ the charge density and $\varepsilon_0$ the dielectric constant.

- No magnetic monopole (no particle from which magnetic field lines radiate)

\[
\nabla \cdot \mathbf{B} = 0
\]
Ohm’s law

- Relates current density $\mathbf{j}$ to electric field $\mathbf{E}$.
- In a material at rest, we assume the simple form
  \[
  \mathbf{j} = \sigma \mathbf{E}
  \]
  with $\sigma$ the electrical conductivity.
- In the reference frame moving with the fluid, the electric, magnetic fields and current become
  \[
  \mathbf{E}' = \mathbf{E} + \mathbf{u} \times \mathbf{B}, \quad \mathbf{B}' = \mathbf{B}, \quad \mathbf{j}' = \mathbf{j}
  \]
- In the original reference, Ohm’s law is
  \[
  \mathbf{j} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B})
  \]
A solid electrically conducting disk rotates about an axis.

- Uniform magnetic field $\mathbf{B}_0$ aligned with the rotation axis.
- The electromotive force $q(\mathbf{u} \times \mathbf{B}_0)$ separates the charges.

Dynamo: conversion of kinetic energy into magnetic energy.

If the disc rotation rate exceeds a critical value, $\mathbf{B}_0$ can be switched off and the dynamo will continue to operate: the dynamo has become self-excited.
Homopolar disc dynamo

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- A conducting wire is wound around the disc and joins the rim and the axis: electric current flows in the wire and across the disc.
- Winding is such that the induced magnetic field $B$ reinforces the applied magnetic field $B_0$. 
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Now using Faraday’s and Ampère’s laws, we get

\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B - \eta \nabla \times B), \]

where \( u \) is the fluid velocity, and \( \eta = 1/(\mu_0 \sigma) \) is the magnetic diffusivity.
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If \( \eta \) is constant, then

\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B) + \eta \nabla^2 B \]

where we used \( \nabla \cdot B = 0 \).
For an incompressible fluid ($\nabla \cdot u = 0$) we can re-write the magnetic induction equation as

$$\frac{DB}{Dt} = (B \cdot \nabla)u + \eta \nabla^2 B$$

where the material derivative is

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u \cdot \nabla$$

- Induction is produced by the shear of magnetic field lines.
- The induction equation is linear in $B$ and the relative importance of each term is independent of the field strength.
- Magnetic Reynolds number:

$$\frac{|\nabla \times (u \times B)|}{|\eta \nabla^2 B|} \sim \frac{UL}{\eta} \left( = \frac{L^2/\eta}{L/U} = \frac{\text{magnetic diffusion timescale}}{\text{turnover timescale}} \right)$$

$$Rm \equiv \frac{UL}{\eta}$$

- Magnetic diffusion is dominant on small scales ($Rm \ll 1$) and negligible on large scales ($Rm \gg 1$).
Frozen flux limit: $Rm \to \infty$

Neglecting diffusion on the large scales:

$$\frac{DB}{Dt} = (B \cdot \nabla)u.$$ 

- Magnetic field lines behave as material lines and move with the fluid.
- The magnetic flux through a material surface is conserved.

$$\int\int_S B \cdot dS = \text{const.}$$

This is Alfvén’s theorem: Magnetic flux is “frozen” into the fluid.

- If the surface expands then the field must become weaker, so that the total flux is unchanged.
- Alfvén theorem no longer holds exactly if there is diffusion: reconnection is possible and field lines are no longer material curves.
\( \nabla \cdot B = 0 \) implies that only two independent scalar fields are needed to specify \( B \).

In spherical geometry:

\[
B = B_T + B_P, \quad B_T = \nabla \times T, \quad B_P = \nabla \times \nabla \times P.
\]

\( T \): toroidal component, \( P \): poloidal component.

\[
\begin{align*}
B_r &= \frac{1}{r} L^2(P), \\
B_\theta &= \frac{1}{\sin \theta} \frac{\partial T}{\partial \phi} + \frac{1}{r} \frac{\partial}{\partial \theta} \frac{\partial rP}{\partial r}, \\
B_\phi &= -\frac{\partial T}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \frac{\partial rP}{\partial r},
\end{align*}
\]

with

\[
L^2 = \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} - r^2 \nabla^2 = -\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} - \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial^2 \phi}.
\]

- Radial component of the induction equation gives the equation for \( P \) (\( L^2P = r \cdot B \)).
- Curl of the induction equation gives the equation for \( T \) (\( L^2T = r \cdot \nabla \times B \)).
- Radial component of the diffusion term and its curl can be separated into poloidal and toroidal parts. Only coupling arises from the induction term.
Free decay modes: $Rm \to 0$

- Consider a conducting sphere of radius $a$ surrounded by an electrically insulating region.
- For $r < a$, 
  \[ \frac{\partial B}{\partial t} = \eta \nabla^2 B, \]
- We look for solutions of the form $B = B_0(r, \theta, \phi)e^{-\sigma t}$ where $\sigma$ is the decay rate.
- We must solved the problem: $(\nabla^2 + \sigma/\eta)B_0 = 0$.
- Spherical harmonics expansion:
  \[ T = \sum_{l=0}^{\infty} \sum_{m=0}^{l} t^m_l(r)Y^m_l(\theta, \phi), \quad P = \sum_{l=0}^{\infty} \sum_{m=0}^{l} p^m_l(r)Y^m_l(\theta, \phi). \]
- This greatly simplifies the $L^2$ operator: $L^2(Y^m_l) = l(l + 1)Y^m_l$.
- For each spherical harmonics, we have to solve:
  \[ \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial p^l_l}{\partial r} + \left( \frac{\sigma}{\eta} - \frac{l(l+1)}{r^2} \right) p^l_l = 0 \]
- The solution of this equation is a spherical Bessel function $j_l$. 
Free decay toroidal modes

- Outside the sphere ($r > a$): $j = 0$.
- For the toroidal component at $r > a$:

$$j_r = \frac{1}{\mu_0} r \cdot \nabla \times \mathbf{B} = \frac{1}{\mu_0} L^2 T = 0 \quad \rightarrow \quad T = 0.$$

- The toroidal decay solution for $r < a$ is $t^m_l = j_l(\sqrt{\sigma/\eta})$.
- $T$ is continuous at $r = a$ so matching the solutions at $r = a$ gives $\sigma_l = \eta x_l^2 / a^2$, with $x_l$ the lowest zero of $j_l$ (for $l = 1$, $x_1 = 4.493$).

- The decay time for the spherical harmonic of degree $l$ is $\tau_l = a^2 / \eta x_l^2$.
- In the Earth's core ($a = 3500$ km, $\eta = 1$ m$^2$/s), the toroidal mode $l = 1$ decays in about 20000 yrs.
- Toroidal modes of larger $l$ decay faster.
Free decay poloidal modes

- For the poloidal component at $r > a$: $j_T = -\nabla^2 P / \mu_0 = 0$ ($j_T$: toroidal current).
- At $r > a$: 
  
  $$p_l^m = \frac{A}{r^{l+1}}, \quad A = \text{const.}$$

  and so,

  $$\frac{\partial p_l^m}{\partial r} + \frac{l + 1}{r} p_l^m = 0.$$  

- The poloidal decay solution for $r < a$ is $p_l^m = j_l(\sqrt{\sigma \eta} r)$.
- $P$ and $\partial P / \partial r$ are continuous at $r = a$ so matching the solutions at $r = a$ and using the Bessel functions recurrence relations gives $\sigma_l = \eta x_{l-1}^2 / a^2$.

- For the dipole $l = 1$, the lowest zero is $x_0 = \pi$ so the decay time for the dipole is $\tau = a^2 / \eta \pi^2$.
- In the Earth’s core, the dipole decays in about 40000 yrs.
- Poloidal modes with larger $l$ decay faster.
The turnover timescale is

$$\tau_U = \frac{R}{U} \approx \frac{10^6 \text{m}}{5 \times 10^{-4} \text{mm/s}} = 60 \text{ yr}$$

$$Rm = \frac{\tau_\eta}{\tau_U} \approx \frac{40000 \text{ yr}}{60 \text{ yr}} \approx 600$$

Frozen flux approximation in Earth’s core?
Core flow inversion
Cowling’s anti-dynamo theorem

Cowling (1934)

Axisymmetric magnetic fields cannot be maintained by dynamo action.

Important note:
- This theorem disallows axisymmetric $B$, but not axisymmetric $u$.
- An axisymmetric $u$ can create a non-axisymmetric $B$. See examples of the Ponomarenko dynamo and Dudley & James dynamo.
What about Saturn’s magnetic field then?

Saturn’s magnetic field measured outside of the planet is remarkably axisymmetric...

Possible scenario: a stably-stratified layer at the top of the core screens the non-axisymmetric field produced in the core?
Assuming that the magnetic field and the velocity are axisymmetric (a non-axisymmetric flow always create a non-axisymmetric field):

\[
\begin{align*}
  \mathbf{u} &= \bar{u}_\phi \mathbf{e}_\phi + \mathbf{u}_p, \\
  \mathbf{B} &= \bar{B}_\phi \mathbf{e}_\phi + \mathbf{B}_p = \bar{B}_\phi \mathbf{e}_\phi + \nabla \times (A \mathbf{e}_\phi),
\end{align*}
\]

where \(A \mathbf{e}_\phi\) is the vector potential.

The induction equation now becomes

\[
\begin{align*}
  \frac{\partial A}{\partial t} + \frac{1}{s} (\bar{u}_p \cdot \nabla) (sA) &= \eta \left( \nabla^2 - \frac{1}{s^2} \right) A, \\
  \frac{\partial \bar{B}_\phi}{\partial t} + s (\bar{u}_p \cdot \nabla) \frac{\bar{B}_\phi}{s} &= \eta \left( \nabla^2 - \frac{1}{s^2} \right) \bar{B}_\phi + s \mathbf{B}_p \cdot \nabla \left( \frac{\bar{u}_\phi}{s} \right),
\end{align*}
\]

where \(s = r \sin \theta\).

- Both equations have an advection term and a diffusion term.
- The azimuthal field has a source term: shearing of the poloidal magnetic field lines by the gradients of the angular velocity \(\bar{u}_\phi / s\).
- The poloidal field has no source term so it will just decay. A source term can only be provided by non-axisymmetric terms.
The kinematic dynamo problem

**Kinematic** problem:
- The velocity \( u \) is a given function of space (and possibly time).
- The problem is linear in \( B \).
- In the simple case where \( u \) is independent of time, we look for solutions
  \[
  B = B_0(x, y, z) e^{pt},
  \]
  \[p = \sigma + i \omega\text{ with growth rate } \sigma \text{ and frequency } \omega.
  \]

\[
\frac{\partial B}{\partial t} = \nabla \times (u \times B) + \frac{1}{Rm} \nabla^2 B,
\]

\[
pB_0 = \nabla \times (u \times B_0) + \frac{1}{Rm} \nabla^2 B_0.
\]
- We compute the growth rate as a function of the control parameter \( Rm \): positive growth rate \( \rightarrow \) dynamo.

**Dynamic** (or self-consistent) problem:
- \( u \) is solved using the Navier-Stokes equation.
- \( B \) changes \( u \) through the Lorentz force: the dynamo stops to grow and the amplitude of the magnetic field saturates.
Kinematic dynamos: disc dynamo
An example of a kinematic dynamo: Ponomarenko dynamo

Ponomarenko (1973)

- Conductor fills all space.
- No flow outside a cylinder of radius $a$.
- Inside the cylinder: helical flow
  \[ \mathbf{u} = s\Omega \mathbf{e}_\phi + U\mathbf{e}_z, \]
  with $\Omega$ and $U$ constant.

Note:
- Anti-dynamo theorem: no dynamo can be maintained by a planar flow (i.e. a flow with only 2 components) (Zeldovich 1957).
- So if $U = 0$: the dynamo fails.
An example of a kinematic dynamo: Ponomarenko dynamo

- The control parameters are the magnetic Reynolds number as
  \[ Rm = \frac{a u_{\text{max}}}{\eta}, \quad \text{with} \quad u_{\text{max}} = \sqrt{U^2 + \Omega^2 a^2}, \]
  and the pitch angle of the spiral: \( \chi = U / (a\Omega) \).
- \( u \) depends only on \( s \), so the solution simplifies to
  \[ B = B_0(s) \exp(i m \phi + i k z + p t) \]
- Substitute in the induction equation to get the ordinary differential equations for \( B_1(s) \) (can be solved analytically).
- Leads to a dispersion relation \( f(p, m, k, Rm, \chi) = 0 \).
- The smallest value of \( Rm \) for which \( \Re(p) = 0 \) is minimised over \( m \) and \( k \) for a given \( \chi \rightarrow \text{critical} Rm \) for the onset of dynamo action.

\( Rm_c \approx 18 \): moderate \( Rm \) that can be reached in a laboratory experiment with liquid sodium.

Surfaces of constant magnetic field amplitude showing the spiralling field following the flow spiral.
FIG. 6. The main part of the Riga dynamo facility: (1) Propeller moved via belts by two motors (not shown). (2) Helical flow region without any flowguides; flow rotation is maintained by inertia only. (3) Back-flow region. (4) Sodium at rest. (5) Thermal insulation. F: Position of the flux-gate sensor. H1–H6: Positions of six vertically aligned Hall sensors.
**Ω-effect**: Differential rotation shears poloidal magnetic field lines to generate a toroidal magnetic field.
Parker loop mechanism: Magnetic field lines are bent and twisted by a cyclonic eddy. This creates an electric current parallel to the original field. Poloidal field can be created out of toroidal field with this mechanism.
Conceptual dynamo mechanisms

Stretch-Twist-Fold:

- Loop of flux is stretched twice its length.
- Alfvén theorem: cross-section of the tube is halved $\rightarrow |B|$ doubles.
- Twist the loop and fold it.
- Large gradients at R leads to larger diffusion there and reconnection.
- Each new loop has the same flux as the original tube: total flux has doubled.
Mean field dynamo theory

- We partition the velocity and magnetic field into averaged and fluctuating parts:
  \[ u = \bar{u} + u', \quad B = \bar{B} + B', \]
  where \( \bar{\cdot} \) represents a time/space/ensemble average and \( \bar{B}' = \bar{u}' = 0 \).
- We assume that averaging commutes with differentiating: \( \nabla \times \bar{B} = \nabla \times \bar{B} \) for instance.
- We average the induction equation:
  \[
  \frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{u} \times \bar{B}) + \eta \nabla^2 \bar{B}.
  \]
  \[
  \frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{u} \times \bar{B}) + \nabla \times \mathcal{E} + \eta \nabla^2 \bar{B}.
  \]
  where \( \mathcal{E} = \bar{u}' \times \bar{B}' \) is the mean e.m.f. (\( \bar{u} \times \bar{B}' = \bar{u}' \times \bar{B} = 0 \)).
- Cowling’s theorem is avoided if \( \mathcal{E} \neq 0 \).
- To make further progress, we must adopt a mean-field closure. The simplest is the \( \alpha \) model: \( \mathcal{E} = \alpha \bar{B} \) (Steenbeck, Krause & Rädler, 1966).
- The \( \alpha \)-effect can be understood as an averaged Parker loop mechanism.
- The \( \alpha \)-effect is correlated to the helicity \( (u' \cdot (\nabla \times u')) \) for low-amplitude or highly fluctuating turbulence.
- Mean-field models can be tuned to match the observations...
Validity of mean field dynamo theory

- In the $\alpha$ model, we assume that $B'$ is proportional to $\overline{B}$, and so $\overline{B} = 0$ implies that $B' = 0$. Is this always true?

- Equation for the fluctuating field:

$$\frac{\partial B'}{\partial t} = \nabla \times (\overline{u} \times B') + \nabla \times (u' \times \overline{B}) + \nabla \times (u' \times B') + \nabla \times (u' \times B') + \eta \nabla^2 B'$$

- The $\alpha$-model assumes that there is no small-scale dynamo: $Rm \ll 1$ at small scales.

- In the Solar convective zone, $Rm$ is huge on the large scale ($\sim 10^{11}$)!

- The assumptions used for the averaging require a sufficient length scale separation between the mean and the fluctuating parts.
Another example of a kinematic dynamo: G.O. Roberts dynamo

Roberts (1972)

\[
\begin{align*}
\mathbf{u} &= (\cos y, \sin x, \sin y + \cos x) \\
\mathbf{\omega} &= \nabla \times \mathbf{u} = \mathbf{u} \\
H &= \mathbf{u} \cdot \mathbf{\omega} = |\mathbf{u}|^2 > 0
\end{align*}
\]

- \(\mathbf{u}\) is independent of \(z\), but has 3 components (avoid the planar flow anti-dynamo theorem).
- Ponomarenko dynamo has a single roll and \(\mathbf{B}\) is on the scale of the roll: model for a small-scale dynamo where the lengthscale of \(\mathbf{B}\) is comparable with the lengthscale of \(\mathbf{u}\).
- G.O Roberts dynamo: collection of rolls so the field can be coherent across many rolls: \(\mathbf{B}\) can be large scale.
Another example of a kinematic dynamo: G.O. Roberts dynamo

Roberts (1972)

\[ \mathbf{B} = \mathbf{B}_0(x, y) \exp(pt + ikz) \]

- Anti-dynamo theorem implies that \( k \neq 0 \) (Cowling’s theorem in planar geometry: a field independent of \( z \) cannot be maintained by dynamo action).
52 tubes inside a cylinder of diameter 1.7 m and height 0.7 m.

The scale separation between individual tubes and the whole apparatus is about 10.

Produce a large-scale magnetic field.

Good agreement between the dynamo onset in the experiment and kinematic calculations.
Dynamo experiments with less constrained flows

Von Kármán Sodium experiment (Cadarache, France)
Monchaux et al. (2007)

- 2 impellers (radius: 15 cm) positioned 37 cm apart.
- 50 L of liquid Sodium.
- Dynamo for $Rm \sim 50$, but only when the impellers are made of soft iron (high magnetic permability).
- $\alpha$-effect from the spiraling flow produced between the blades of the impellers? (e.g. Laguerre et al., 2008)
Dynamo experiments with less constrained flows

Von Kármán Sodium experiment (Cadarache, France)

\[ \Theta = \frac{F_1 - F_2}{F_1 + F_2} \]

Monchaux et al. (2009) \( \Theta = (F_1 - F_2)/(F_1 + F_2) \), blue: \( B_x \), red: \( B_y \) and green: \( B_z \)
Dynamo experiments with less constrained flows

- $Rm = RePm$ where the magnetic Prandtl number is $Pm = \nu / \eta$ ($\nu =$ fluid viscosity)
- $Pm \approx 5 \times 10^{-6}$ for liquid Sodium $\rightarrow Re \geq 10^7$ for dynamo action in experiment.
- Without a precise control of the flow, the effects of the turbulence can be to increase the critical $Rm$ of the dynamo onset.
- More turbulent experiment have not yet reached the critical $Rm$.

Spherical Couette flow (Lathrop et al., Maryland)
$Rm \approx 700$, but no dynamo so far.
Self-consistent dynamo simulations

Dimensionless governing equations:

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{2}{E_k} \mathbf{e}_z \times \mathbf{u} = -\nabla P + \nabla^2 \mathbf{u} + \frac{Ra}{Pr} T e_r + (\nabla \times \mathbf{B}) \times \mathbf{B},
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{P_m} \nabla^2 \mathbf{B},
\]

\[
\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{Pr} \nabla^2 T,
\]

\[
\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{B} = 0.
\]

- **Coriolis force**, **Buoyancy driving**, **Lorentz force**.
- Ekman number: \( Ek = \nu / (\Omega d^2) \),
- Rayleigh number: \( Ra = \alpha g \Delta T d^3 / (\nu \kappa) \),
- magnetic Prandtl number: \( Pm = \nu / \eta \),
- Prandtl number: \( Pr = \nu / \kappa \),
- with \( \Omega \) the rotation rate, \( \nu \) the viscosity, \( d \) the size of the domain, \( \kappa \) the thermal conductivity, and \( \eta \) the magnetic diffusivity.
Self-consistent dynamo simulations – First success in the 90’s

Glatzmaier & Roberts (1995)
Christensen et al. (2001), $Ek = 10^{-3}$

a) $B_r$ at the outer radius $r_o$, b) $u_r$ at $r = 0.8r_o$, c) axisymmetric $\mathbf{B}$, (colour: toroidal field, field lines: poloidal field), d) axisymmetric $\mathbf{u}$ (colour: zonal flow, streamlines: meridional flow).
Generation of a large-scale magnetic field by the convective flows

Aubert (2003)

Christensen & Wicht (2015)
Exploration of the parameter space

Schaeffer et al. 2017
Exploration of the parameter space

Christensen & Aubert (2006)
Reversals in dynamo simulations occurs when
\[ Ro_\ell = \frac{U}{\Omega \ell} \approx 0.1 \]
This implies an important role of inertia at the scale where the dynamo operates.
In the Earth’s core: \( Ro_\ell \approx 0.1 \) for \( \ell \approx 10m \).
\[ Rm_\ell = \frac{U\ell}{\eta} \ll 1 \] for \( \ell \approx 10m \).
\( \alpha \)-effect could operate at these small scales but requires well correlated flows...

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- \( \alpha \)-effect could operate at these small scales but requires well correlated flows...

Christensen & Aubert (2006)
\( f_{dip} \) = relative dipole strength
Recent dynamo simulations: feedback of the magnetic field

Schaeffer et al. 2017

\[ A = \left( \frac{E_k}{E_m} \right)^{1/2} \]
Recent dynamo simulations: feedback of the magnetic field

Yadav et al. (2016): Effect of the field on the lengthscale is more pronounced at low Ek
Recent dynamo simulations: feedback of the magnetic field

Movie: Schaeffer et al. (2017) \( E_k = 10^{-7} \)